JIGNASA

STUDENT STUDY PROJECT – 2019-20

SRNK GOVT DEGREE COLLEGE BANSWADA, KAMAREDDY

SEQUENCES AND SERIES DEPTOFMATHEMATICS

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INTRODUCTION :

In mathematics, a sequence is a list of objects (or events) which have been ordered in a sequential fashion; such that each member either comes before, or after, every other member. ... A series is a sum of a sequence of terms. That is, a series is a list of numbers with addition operations between them.

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant.

Imagine yourself at a pizza hut. You have placed an order and your order number is 282. So currently they are serving the order number 275. So how many orders do you think will be served before your number? Yes, six orders more because you are in a sequence. To understand this better, let us learn about the sequences and series.

Example

2,4,6,8,10….is an arithmetic sequence with the common difference 2.

If the first term of an arithmetic sequence is a_1 and the common difference is d, then the nth term of the sequence is given by:

$$an=a1+(n-1)d$$

An arithmetic series is the sum of an arithmetic sequence. We find the sum by adding the first, a_1 and last term, a_n, divide by 2 in order to get the mean of the two values and then multiply by the number of values, n:

$$Sn=n2(a1+an)$$

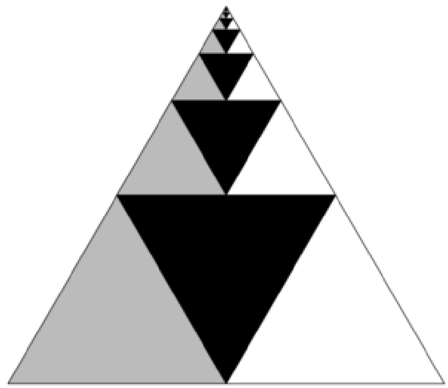
Example

Find the sum of the following arithmetic series 1,2,3…..99,100

We have a total of 100 values, hence n=100. Our first value is 1 and our last is 100. We plug these values into our formula and get:

$S100=1002(1+100)=5050$

GEOMETRIC PROGRESSION



AIMS AND OBJECTIVES :

- Define sequences and identify the different kinds of sequences
- Find the nth term or the general term of a sequence for which some intial terms are given
- Find the common difference of an arithmetic sequence.
- Find the common ratio of a geometric sequence.
- Find arithmetic means, harmonic means and geometric means.
- Find the sum of a finite arithmetic series, harmonic series and geometric series.
- Find the sum of an infinite geometric series.
- Apaply test for convergence / divergence of series.
- Apply cauchyconvergence test for sequences and series.

METHODOLOGY : PROJECT METHOD

• Introduction to sequences and series

• sequences and series in sequences containing the series of the one or more

Geometric and arithmetic series with request to one or more geometric progress.

WORKED EXAMPLES

SEQUENCES

A sequence $\{ a_{n} \}$ is an infinite list of numbers $$a_{1}, a_{2}, a_{3}, \ldots,$$ where we have one number a_{n} for every positive integer n.

Defining sequences

A sequence is a list of numbers in a special order. It is a string of numbers following a particular pattern, and all the elements of a sequence are called its terms. There are various types of sequences which are universally accepted, but the one which we are going to learn right now is the arithmetic progression

Pattern. We can specify it by listing some elements and implying that the pattern shown continues.

Example

For example $$2, 4, 6, 8, \ldots$$ would be the sequence consisting of the even positive integers.

Formula. We can also specify a sequence by giving a formula for the term that corresponds to the integer n.

Example

For example the sequence $$2, 4, 6, 8, \ldots$$ can also be specified by the explicit formula $$a_{n} = 2n.$$ **Recursively.** Finally, we can also provide a rule for producing the next term of a sequence from the previous ones. This is called a *recursively* defined sequence.

Example

For example the sequence $$2, 4, 6, 8, \ldots$$ can be specified by the rule $$a_{1} = 2 \quad \text{ and } \quad a_{n} = a_{n-1} +2 \text{ for } n\geq 2.$$ This rule says that we get the next term by taking the previous term and adding 2. Since we start at the number 2 we get all the even positive integers.

Let's discuss these ways of defining sequences in more detail, and take a look at some examples.

Part 1: Arithmetic Sequences

The sequence we saw in the previous paragraph is an example of what's called an *arithmetic* sequence: each term is obtained by **adding a fixed number** to the previous term.

Alternatively, the difference between consecutive terms is always the same.

Arithmetic Sequence and Series

An arithmetic sequence is a sequence of numbers such that the difference d between each consecutive term is a constant.

$$a, a+d, a+2d, a+3d, \ldots$$

The n^{th} term, $a_n = a + (n-1)d$

Sum of first n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_n = \frac{n}{2}[a + a_n]$$

Finding the twelfth term (a_{12})	Finding the eighty-second term (a_{82})
$a_n = 43 + (n-1)\cdot(-3)$	$a_n = 43 + (n-1)\cdot(-3)$
$a_{12} = 43 + (12-1)\cdot(-3)$	$a_{82} = 43 + (82-1)\cdot(-3)$
$= 43 + (11)\cdot(-3)$	$= 43 + (81)\cdot(-3)$
$= 43 + (-33)$	$= 43 + (-243)$
$a_{12} = 10$ ✓	$a_{82} = -200$ ✓

General Formula.

If a sequence a_{n} is arithmetic, then there is a fixed number d so that $a_{n+1} -a_{n} =d$ for any $n.$ The number d is usually called the **step** or **difference**. Let's try to find a formula for the term a_{n} of an arithmetic sequence in terms of d and a_{1}.

Let's start with $a_{n} = a_{n-1} + d$. Applying this again, we see that since $a_{n-1} = a_{n-2} + d$ so we get that $a_{n} = a_{n-2} + d +d = a_{n-2} + 2d$. We can continue this way and get: \begin{align*} a_{n} &= a_{n} = a_{n-1} + d \\ &= a_{n-2} + d +d = a_{n-2} + 2d \\ &= a_{n-3} + d + d = a_{n-3} + 3d \\ &\vdots \\ &= a_{2} + (n-2)d \\ &= a_{1} + (n-1)d \\ \end{align*} So we get that in an arithmetic sequence a_{n} with steps of size d, the formula for a_{n} is given by: $$a_{n} = a_{1} + (n-1)d$$

Example. Consider the sequence $3, 8, 13, 18, 23, 28, \ldots$. Is it arithmetic? If so, find a formula for a_{n}, and use it to find a_{101}, the 101st term in the sequence.

Solution. This sequence is arithmetic, since the difference between each term is $5.$ ($8-3 = 13-8 = 18-13 = \cdots = 5$.) So this is an arithmetic sequence with step $d=5$ and first term $a_{1} = 3$.

Our formula above gives $a_{n} = a_{1} + (n-1)d = 3 + (n-1)5$.
For a_{101} we plug in $n=101$ into this formula to obtain $a_{101} = 3 + (100)5 = 503$.

Part 2: Geometric Sequences

Consider the sequence $2, 4, 8, 16, 32, 64, \ldots$. This sequence is not arithmetic, since the difference between terms is not always the same. If we look closely, we will see that we obtain the next term in the sequence by **multiplying the previous term by the same number**. Equivalently, the ratio of consecutive terms is always the same (namely 2).

A sequence a_{n} where there is a fixed r so that $\frac{a_{n}}{a_{n-1}} = r$ for all n is called a geometric sequence. The number r is usually called the **ratio**.

Geometric Progression

Let us carry out an activity. Take a paper and fold it as many times as you can. So how many times did you fold the paper? Maybe four to five times right? Now can you calculate the height of the stack of the paper after it has been folded number of times? How do you calculate it? The answer to this is geometric progression

Examples

$$\sum_{n=0}^{\infty} 3\cdot(1)^{n} \qquad \sum_{n=0}^{\infty}(-1)^{n} \qquad \sum_{n=0}^{\infty}(2)^{-n} \qquad \sum_{n=2}^{\infty}\left(\frac{1}{3}\right)^{n}$$

$$\sum_{n=1}^{\infty}\left(\frac{-1}{5}\right)^{n} \qquad \sum_{n=1}^{\infty}(0.6)^{n-1} \qquad \sum_{n=1}^{\infty}\sqrt[n]{2}$$

$$\sum_{n=0}^{\infty}\frac{3^{n}+2^{n}}{6^{n}} \qquad \sum_{n=1}^{\infty}\ln\left(\frac{n}{2n+5}\right)$$

Geometric Series

$$S_N = \sum_{n=0}^{N} c\cdot r^{n}$$

$$S_N = c + c\cdot r + c\cdot r^{2} + c\cdot r^{3} + \ldots + c\cdot r^{N} \qquad (1)$$

If we multiply both sides by r we get

$$r\cdot S_N = c\cdot r + c\cdot r^{2} + c\cdot r^{3} + c\cdot r^{4} + \ldots + c\cdot r^{N+1} \qquad (2)$$

If we subtract (2) from (1), we get

$$\left.\begin{array}{l} S_N - r\cdot S_N = c - c\cdot r^{N+1} \\ S_N(1-r) = c(1-r^{N+1}) \end{array}\right\} \Rightarrow S_N = \frac{c(1-r^{N+1})}{1-r}$$

General Formula:

Let's try to find the formula for the term a_{n} of a geometric sequence in terms of r and the first term.

Let's start with the relation $\frac{a_{n}}{a_{n-1}} = r$. This gives $a_{n} = r a_{n-1}$. Using this again, we get $a_{n} = r (ra_{n-2}) = r^{2} a_{n-2}$. We can continuer this way and get: \begin{align*} a_{n} &= r a_{n-1} \\ &= r^{2} a_{n-2} \\ &= r^{3} a_{n-3} \\ &\vdots \\ &= r^{n-2} a_{2} \\ &= r^{n-1} a_{1} \\ \end{align*} So we get that for a geometric sequence a_{n} with ratio r, the formula for a_{n} is given by: $$a_{n} =r^{n-1} a_{1}$$

Example.

Consider the geometric sequence $3, 6, 12, 24, 48, \ldots$. Find a formula for a_{n} and use it to find a_{7}.

Solution. To find r, we should look at the ratio between successive terms: $r= \frac{a_{1}}{a_{2}} =\frac{6}{3} = 2$. Then using the formula above we get $a_{n} = r^{n-1} a_{1} = 2^{n-1} \cdot 3$.

To find a_{7} we set $n=7$ and get $a_{7} = 2^{7-1} \cdot 3 = 2^{6} \cdot 3 = 64 \cdot 3 = 192$.

Part 3: Recursive Sequences

We have already briefly discussed this idea in the first paragraph. We shall now discuss this in more detail, together with some extra examples.
As we saw in the section on geometric sequences, we can define a geometric sequence either by the rule $a_{n} = r^{n-1} a$, or by the rule that $a_{n} = r a_{n-1}$.
The latter rule is an example of a *recursive* rule. A *recursively defined sequence*, is one where the rule for producing the next term in the sequence is written down explicitly in terms of the previous terms.

Let's consider the following (rather famous) example.

Example.

Define a sequence a_{n} as follows: Let $$a_{1} = 1 \quad , \quad a_{2} = 1 \quad \text{, and} \quad a_{n} = a_{n-1} + a_{n-2} \text{ for } n\geq 2.$$ This rule says that to get the next term in the sequence, you should add the previous two terms. Since this rule requires two previous terms, we need to specify the first two terms of the sequence a_{1}, a_{2} to get us started. Using this we can start to list the terms in the sequence, and get $1, 1, 2, 3, 5, 8, 13, 21, 34,\ldots$. (This is the well known **Fibonacci** sequence.)

Example.

Consider the recursively defined sequence $$a_{1} = 1 \quad , \quad a_{2} = 1 \quad , \quad a_{3} = 1 \quad \text{, and} \quad a_{n} = \frac{a_{n-3}}{a_{n-1} + a_{n-2}} \text{ for } n\geq 3.$$ List the first 7 terms of this sequence.

Solution. \begin{align*} a_{1} &= 1 \\ a_{2} &= 1 \\ a_{3} &= 1 \\ a_{4} &= \frac{1}{1+1} = \frac{1}{2} \\ a_{5} &=\frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3} \\ a_{6} &= \frac{1}{\frac{1}{2} + \frac{2}{3}} = \frac{1}{\frac{7}{6}} = \frac{6}{7} \\ a_{7} &= \frac{ \frac{1}{2}}{\frac{2}{3} + \frac{6}{7}} = \frac{ \frac{1}{2}}{\frac{32}{21}} = \frac{21}{64} \end{align*}

Part 4: Sequences via Lists

The method of using a list to specify a sequence perhaps is the most tricky, since it requires us to look at a short piece of a sequence, and guess at the pattern or rule that is being used to produce the terms in the sequence.
Now that we have seen some more examples of sequences we can discuss how to look for patterns and figure out given a list, how to find the sequence in question.

Example.

When given a list, such as $1, 3, 9, 27, 81, \ldots$ we can try to look for a pattern in a few ways. Now that we have seen **arithmetic, geometric and recursive** sequences, one thing we can do is try to check if the given sequence is one of these types.

Arithmetic? To check if a sequence is arithmetic, we check whether or not the difference of consecutive terms is always the same. In this case, the difference changes: $$a_2- a_1 = 3-1 =2 \neq 6 = 9-3 = a_3-a_2.$$ **Geometric?** To check if a sequence is geometric we check whether or not the ratio of consecutive terms is always the same. In the case it is, so we conclude that the sequence is geometric: $$\frac{3}{1}= \frac{9}{3} = \frac{27}{9} = \frac{81}{27} = 3.$$ This tells us that the sequence is geometric with ratio 3, and initial term 1, so we get that the sequence is given by $$a_{n} = 3^{n-1}.$$ This sequence can also be defined recursively, by the formula $$a_{1} =1 \quad \text{, and} \quad a_{n} = 3a_{n-1} \text{ for } n\geq 2.$$

Example.

Consider the sequence $1, -3, -7, -11, -15, -19, -23, \ldots$. Determine a formula for the n^{th} term in the sequence.

Solution. We quickly see that this series is not geometric, since $\frac{1}{-3} \neq \frac{-3}{-7}$.
We can now try to see if the sequence is arithmetic. If we look at the differences of consecutive terms, we get: $-3 - 1 = -4 = -7 - (-3) = -11 - (-7)$, so we see that this is an arithmetic sequence with difference $d=-4$. So the general term is $$a_{n} = a_{1} + (n-1) d = 1 +(n-1) (-4) = - 4n + 5.$$
(We could also try to identify a recursive definition of this sequence.)

WHY DOES THE FORMULA WORK?

Let's see **why** the formula works, because we get to use an interesting "trick" which is worth knowing.

First, we will call the whole sum **"S"**:

$S = a + (a + d) + ... + (a + (n−2)d) + (a + (n−1)d)$

Next, rewrite S in reverse order:

$S = (a + (n−1)d) + (a + (n−2)d) + ... + (a + d) + a$

Now add those two, term by term:

$$\begin{array}{llllllllll} S & = & a & + & (a+d) & + & \ldots & + & (a + (n-2)d) & + & (a + (n-1)d) \\ S & = & (a + (n-1)d) & + & (a + (n-2)d) & + & \ldots & + & (a + d) & + & a \\ \hline 2S & = & (2a + (n-1)d) & + & (2a + (n-1)d) & + & \ldots & + & (2a + (n-1)d) & + & (2a + (n-1)d) \end{array}$$

Each term is the same! And there are "n" of them so ...

$2S = n \times (2a + (n-1)d)$

Now, just divide by 2 and we get:

$$S = (n/2) \times (2a + (n-1)d)$$

Which is our formula:

$$\sum_{k=0}^{n-1}(a + kd) = \frac{n}{2}(2a + (n-1)d)$$

CONCLUSION & SUGGESTION :

As you can see, Excel is a very powerful tool to use for the investigation and demostration of sequences and series. You can show your students how to model sequences and series in Excel and then send them off to do their own investigations of sequences and series that may interest them. You students can use Excel to develop proofs of the convergence or divergence of particular sequences or series. I am not saying here that a demonstration of convergence or divergence using Excel is sufficient as a proof. I am only saying that the use of Excel in the investigation may help the student to develop a formal proof by way of the use of the various tests for convergence or divergence given in most basic analysis textbooks and in some calculus and algebra textbooks.

Excel is also a wonderful tool for introducing the concept of sequences and series. It is a very useful teaching tool for mathematics at this level even if it was not intended to be used as such. And, since most personal computers now have Excel or some other spread-sheet software, such as Lotus 1-2-3, installed, there is often no additional cost to the student to be able to use Excel in his work.

REFERENCES :

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