Kakatiya Government College, Hanamkonda.

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Student Study project on
Applications of Differential Equations to Real world systems

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CERTIFICATE

This is to certify that the Project Report entitled "**APPLICATIONS OF DIFFERENTIAL EQUATIONS TO REAL WORLD SYSTEMS",** submitted to the Commissioner of Collegiate Education Hyderabad, for the Best student Project award in **JIGNASA** Competition, was carried out by the following students under my guidance.

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1. Introduction

Many problems in engineering and science can be formulated in terms of differential equations. The formulation of mathematical models is basically to address real world problems which have been one of the most important aspects of applied mathematics. It is often the case that these mathematical models are formulated in terms of equations involving functions as well as their derivatives. Such equations are called differential equations. A differential equation is a mathematical equation for an unknown function of one or several variables that relates the values of the function itself and its derivatives of various orders. If only one independent variable is involved, the equations are called ordinary differential equations, otherwise it is called Partial differential equation. Differential equations arise in many areas of science and technology, specifically whenever a deterministic relation involving some continuously varying quantities and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the motion of a body is described by its position and velocity as the time varies. Newton's laws allow to relate the position, velocity, acceleration and various forces acting on a body and state this relation as a differential equation for the unknown position of the body as a function of time. Differential equations are mathematically studied from several different perspectives, mostly concerned with their solutions as the set of functions that satisfy the equation. Only the simplest differential equations admit solutions given by explicit formulas; however, some properties of solutions of a given differential equation may be determined without finding their exact form. If a self-contained formula for the solution is not available, the solution may be numerically approximated using computers. Referring to the number of its independent variables while differential equation is divided into the types ordinary differential equations and partial differential equations, they can further described by attributes such as linearity and order. When we classify DE by linearity we have linear and non linear differential equation. A first order differential equation is a differential equation which contains no derivatives other than the first derivative and it has an equation of the form.

2. Objectives

- To study some real world problems which are described by first order differential equations.
- To analyze and interpret some real world application problems of differential equations.

3. Methodology

- The work is done by collecting some important information from different sources such as books and internet which are related to the topic.
- The facts and concepts obtained from different sources related to the topic.
- Some first order differential equations of real world problems is studied.
- Some facts and concepts related to the topic is discussed.

4. Applications of Differential Equations to Real World Systems

4.1 Newton's law of Cooling

The rate at which the temperature T (t) changes in a cooling body is proportional to the difference between the temperature of the body and the constant temperature $T(s)$ of the surrounding medium. Symbolically, the rate of change is the derivative and the statement is expressed as

$$\frac{dT}{dt} \propto \left(T - T(\mathrm{s})\right)$$

$$\frac{dT}{dt} = k\left(T - T\left(s\right)\right)$$

$$\frac{dT}{\left(T - T\left(s\right)\right)} = kdt$$

Integrating on both sides, we get

$$\frac{dT}{dt} \propto \left(T - T(\mathrm{s})\right)$$

$$T - T\left(s\right) = e^{kt+c}$$

$$T - T\left(s\right) = e^{kt}.e^{c} \Rightarrow T = T\left(s\right) + Ae^{kt}$$

$$T \; = \; T\left(s\right) + Ae^{kt}$$

Example: Mathematics in Forensic science

A detective discovers a murder victim in a hotel room at 9:00 am one morning. The temperature of the body is 80.0^{o}F. One hour later, at 10:00am, the body has cooled to 75.0^{o} F. The room is kept at a constant temperature of 70.0^{o} F. Assume that the victim had a normal temperature of 98.6^{o} F at the time of death. We will use differential equation to find the time the murder took place.

Let $T\left(t\right)$ be the temperature of the body after t hours. By Newton's Law of Cooling we have the differential equation

$$\frac{dT}{dt} = k\left(T - 70\right)$$

where k is a constant . We will solve the differential equation and get a formula for T .

Step.1 $\dfrac{dT}{(T-70)} = kdt$

Step.2 Integrating on both sides

$\ln(T-70) = kt + c$

We may assume $T - 70 \geq 0$ since the body will never be cooler than the room.

Step.3 Solve for T as a function of t , the function will involve the constants k and c .

$(T-70) = e^{kt+c} = e^c e^{kt} = Ae^{kt}$ where $A = e^c$

Then $T = 70 + Ae^{kt}$ ---(1)

Step.4 Take $t = 0$ when the body was found at 9:00am. Put $t = 0$ and $T = 80.0^o F$ in equation (1) . Then

$80 = 70 + Ae^{k.0} = 70 + A$

$A = 10$ and $T = 70 + 10e^{kt}$

Step.5 After 1 hour, the body temperature is 75.0^{o} F .

Put $t = 1$ and $T = 75.0^o F$ and solve for k .

$75 = 70 + 10e^{k.1}$

$\dfrac{1}{2} = e^k$

$k = \ln\left(\dfrac{1}{2}\right) = -\ln 2$

Thus $T = 70 + 10e^{-t\ln 2}$

Step.6 At what time did the murder take place?

Put $T = 98.6^0 F$ and solve for t.

$98.6 = 70 + 10e^{-t\ln 2}$

$2.86 = e^{-t\ln 2}$

$$t = -\frac{\ln 2.86}{\ln 2} \approx -1.516$$

1.516 hours is about 1 hour and 31 minutes.

The murder took place at 9:00 A.M - 1 hour and 31 minutes = 7:29am.

Example: Suppose that you turn off the heat in your home at night 2 hours before you go to bed, call this time t = 0. If the temperature t = 0 is 66^o F and the time you go to bed (t=2) has dropped 63^o F, What temperature can you expect in the morning, say, 8 hours later(t =10 hours)? Of course this process of cooling off will depend on the outside temperature T(s), which we assume to be constant 32^o F.

Solution: By Newton's law of cooling

$\frac{dT}{dt} = k(T - 32)$

$\frac{dT}{(T-32)} = kdt$

Integrating on both sides

$\ln(T - 32) = kt + c$

$T - 32 = e^{kt+c}$

$T - 32 = e^{kt+c} = e^{kt}e^{\,c} = Ae^{kt}$

Therefore $T(t) = 32 + Ae^{kt}$ --(1)

At t = 0, T (0) = 66 implies $T(0) = 32+A = 66$

Therefore A = 34, substitute A value in equation (1)

$T(t) = 32 + 34e^{kt}$ ---(2)

After 2 hours the temperature is 63 then Put t = 2 in equation (2)

$$T(2) = 32 + 34\,\mathrm{e}^{k.2} = 63$$

$$e^{2k} = \frac{63-32}{34} = 0.911765$$

$$k = \frac{1}{2}\ln 0.911765 = -0.046187$$

We have to find the temperature in the morning i.e., 8 hours later (t =10 hours)

$$T(10) = 32 + 34^{-0.046187} = 53.4^0\,F$$

4.3 Population Growth and Decay

The rate of growth of population is proportional to size of the population at time t

$$\frac{dN(\mathrm{t})}{dt} = kN(\mathrm{t})$$

where $N(t)$ denotes population at time t and k is a constant of proportionality.

$$\frac{dN(\mathrm{t})}{N(\mathrm{t})} = kdt$$

Integrating on both sides

$$\int \frac{dN(\mathrm{t})}{N(\mathrm{t})} = \int kdt$$

We get $ln\,N(t) = kt + lnC$

$$\ln \frac{N(t)}{C} = kt$$

$N(t) = Ce^{kt}$ can be determined if *N(t)* is given at certain time.

Example: The population of a community is known to increase at a rate proportional to the number of people present at a time t. If the population has doubled in 6 years, how long it will take to triple?

Solution : Let *N(t)* denote the population at time *t*. Let *N(0)* denote the initial population (population at t=0).

$$\frac{dN(\mathrm{t})}{N(\mathrm{t})} = kdt$$

$N(t) = Ce^{kt}$ at $t = 0$ we have $C = N(0)$

Given that the population has doubled in 6 years

$$N(6) = Ce^{6k}$$

But $N(6) = Ce^{6k}$

$$e^{6k} = 2$$

$$k = \frac{1}{6} ln2$$

Our question is how long it will take to triple?

$$N(t) = 3N(0) = 3C$$

$$N(0)\, e^{kt} = 3N(0)$$

$3 = e^{\frac{1}{6}(\ln 2)t}$

$\ln 3 = \frac{(\ln 2)t}{6}$

$t = \frac{6\ln 3}{\ln 2}$; 9.6 years

Example: Let population of country be decreasing at the rate proportional to its population. If the population has decreased to 25% in 10 years, how long will it take to be half?

Solution: This phenomenon can be modeled by $\frac{dN}{dt} = kN(t)$

Its solution is

$N(t) = N(0)\, e^{kt}$, where $N(0)$ is the initial population.

Given that population has decreased to 25% in 10 years

$t = 10$, $N(10) = \frac{1}{4}N(0)$

$N(t) = \frac{1}{4}N(0)$

$\frac{1}{4}N(0) = N(0)\, e^{10k}$

$e^{10k} = \frac{1}{4}$

$k = \frac{1}{10}\ln\frac{1}{4}$

Set $N(t) = \frac{1}{4}N(0)$

$N(0)e^{\frac{1}{10}\ln\frac{1}{4}t} = \frac{1}{2}N(0)$

or $t = \dfrac{\ln\frac{1}{2}}{\frac{1}{10}\ln\frac{1}{4}} \simeq$ 8.3 years approximately.

5. Findings

We made an attempt to discuss the application of differential equation by using modeling phenomena of real world problems. Some of the models included are Newton's cooling, population growth and decay and also studied the application of differential equation in physics and Forensic science. From this discussion we get some idea how differential equations are closely associated with science is formulated in terms of differential equations.

As a conclusion many fundamental problems in biological, physical sciences and engineering are described by differential equations. It is believed that many unsolved problems of future technologies will be solved using differential equations. On the other hand, physical problems motivate the development of applied mathematics, and this is especially true for differential equations that help to solve real world problems in the field. Thus, making the study on applications of differential equation and their solutions essential with this regard.

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