JIGNASA STUDENT STUDY PROJECT-2020

“Solution of Partial Differential Equations by Laplace Technique”

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<u>CERTIFICATE</u>

This is to certify that the Project Report entitled "**Solution of Partial Differential Equations by Laplace Technique",** submitted to the Commissioner of Collegiate Education Hyderabad, for the **JIGNASA-2020** Competition, was carried out by the following students under my guidance.

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SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS BY LAPLACE TRANSFORM

1. Introduction

In science, we explore and understand our real world observations, collecting data, finding rules inside or among them, and eventually, we want to explore the truth behind and to apply it to predict the future. This is how we build up our scientific knowledge. The above rules are usually in terms of mathematics. They are called mathematical models. A mathematical model is a description of a system using mathematical concepts and language. The process of developing a mathematical model is termed mathematical modeling. Though equations and graphs are the most common types of mathematical models, there are other types that fall into this category. A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. One important such models is the ordinary differential equations. It describes relations between variables and their derivatives. Such models appear everywhere. For instance, population dynamics in ecology and biology, mechanics of particles in physics, chemical reaction in chemistry, economics, etc. It is therefore important to learn the theory of differential equation, an important tool for mathematical modeling and a basic language of science.

Differential equation: An equation that consists of derivatives is called a differential equation. Differential equations have applications in all areas of science and engineering. Mathematical formulation of most of the physical and engineering problems lead to differential equations.

Differential equations are of two types

- Ordinary differential equation (ODE).
- Partial differential equations (PDE).

Ordinary differential equation: An ordinary differential equation is that in which all the derivatives are with respect to a single independent variable.

Examples

1. $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$, $\frac{dy}{dx}(0)=2, y(0)=4$,

2. $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+5\frac{dy}{dx}+y=\sin x,\ \frac{d^2y}{dx^2}(0)=12,\ \frac{dy}{dx}(0)=2$, $y(0)=4$

Partial Differential Equations

Equation which contain one or more partial derivatives are called partial differential equations. They must, therefore, involve at least two independent variables and one dependent variable. Whenever we consider the case of two independent variables we shall usually take them to be x and y take z to be the dependent variable. The partial deferential coefficients $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}$ will be denoted by p and q respectively. The second order partial derivatives $\frac{\partial^2 z}{\partial x^2},\frac{\partial^2 z}{\partial x\partial y},\frac{\partial^2 z}{\partial y^2}$ are denoted by r,s,t respectively so that $\frac{\partial^2 z}{\partial x^2}=r,\frac{\partial^2 z}{\partial x\partial y}=s,\frac{\partial^2 z}{\partial y^2}=t$

The order of a partial deferential equation is the same as that of the order of the highest partial derivatives in the equation and its degree is the degree of this derivative. For example

1. $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z$ is of first order and first degree
2. $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}=0$ is of second order and first degree.

Formation of Partial differential equations:

Unlike the case of ordinary differential equations which arise from the elimination of arbitrary constants the partial differential equations can be formed either by the elimination of arbitrary constants or by the elimination of arbitrary functions from a relation involving three or more variables.

Elimination of Arbitrary Constants:

Consider z to be a function of two independent variables x and y defined by f(x,y,a,b)=0 …….(1)

In which a and b are two arbitrary constants.

Differentiating (1) partially with respect to x and y we obtain

$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}+p\frac{\partial f}{\partial z}=0 \qquad \text{…..(2)}$$

And

$$\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}\frac{\partial z}{\partial y}=\frac{\partial f}{\partial y}+q\frac{\partial f}{\partial z}=0 \qquad \text{……(3)}$$

By means of the three equations (1),(2) and (3) two constants a and b can be eliminated. This results in partial differential equation of order one in the form

F(x, y, z, p, q) = 0

We illustrate this through few examples:

Ex. Form the partial differential equation by eliminating bthe arbitrary constants a and b from

(a). $ax+by+a^2+b^2$

(b). $ax+by+(\frac{a}{b})-b$

Sol: We have z = ax+by+b^2 ……(1)

Differentiating (1) partially with respect to x and y we have

$$\frac{\partial z}{\partial x}=p=a \qquad \text{…..….(2) and}$$

$$\frac{\partial z}{\partial y}=q=b \qquad \text{………(3)}$$

Putting the values of a and b from these relations in (1) we obtain

Z = px+qy+p^2+q^2

[Notice that to get this partial differential equation we have eliminated a, b from relations

(1), (2) and (3).]

(b). $ax + by + (\frac{a}{b}) - b$

Differentiating (4) partially w. r. t. x and y we get

$$\frac{\partial z}{\partial x} = p = a \text{ and } \frac{\partial z}{\partial y} = q = b$$

Putting the values of a and b from these relations in (1) we get

$$z = px + qy + -q$$

This is the required partial differential equation.

Solutions of a partial differential equation:

Through the earlier discussion examples and exercise problems we can understand that a p.d.e can be formed by eliminating arbitrary constants or arbitrary functions from an equation involving them and three or more variables

Consider a partial differential equation of the form

F(x, y, z ,p, q) = 0 …...(1)

If this is linear in p and q it is called a linear partial differential equation of first order if it is not linear in p, q then it is called a non linear p. d. e of first order

A relation of the type F(x, y, z, a, b) = 0 ……(2)

From which by eliminating a and b we can get the equation (1) is called a complete integral or complete solution of p. d. e (1).

A solution of (1) obtained by giving particular values to a and b in the complete integral (2) is called a particular integral.

If in the complete integral of the form (2) we take f=Ø(a) where a is arbitrary and obtain the envelope of the family of surfaces f(x, y, z, Ø(a)=0.

Then we get a solution containing an arbitrary function. This is called the general integral of (1) corresponding to the complete integral (2).

If in this, we use a definite function Ø(a), we obtain a particular case of the general integral.

If the envelope of the two parameter family of surfaces (2) exists it will also be a solution of (1). It is called a singular integral of the equation (1).

The singular integral differs from the particular integral. It cannot be obtained by giving particular values to the arbitrary constants in (2) where as the particular integral is obtained that way. A more elaborate discussion of these ideas is beyond the scope of this book.

Linear partial differential equations of the first order:

A differential equation involving partial derivatives p and q only and no higher order derivatives is called a first order equation. If p and q occur in the first degree, it is called a linear partial differential equation of first order; otherwise it is called a non linear partial differential equation of the first order. For example $px+qy^2=z$ is a linear p.d.e of first order and $p^2+q^2=1$ is a non linear.

Non linear partial differential equations of first order:

A partial differential equations which involves first order partial derivatives p and q with degree higher than one and the products of p and q is called a non linear partial differential equation.

DEFINITIONS:

1. **Complete Integral**: A solution in which the number of arbitrary constants is equal to the nuber of independent variables is called complete integral or complete solution of the given equation.
2. **Partial Integral**: A solution obtained by giving particular values to the arbitrary constants in the complete integral is called a particular Integral.
3. **Singular Integral**: Let f(x, y, z, p, q)=0 be a partial differential equation whose complete integral is Ø(x, y, z, a, b)=0 …………...(1)

   Differentiating (1) partially w. r. t a and b then equate to zero, we get

   $$\frac{\partial \phi}{\partial a}=0 \quad \text{…………..(2)}$$

   $$\frac{\partial \phi}{\partial b}=0 \quad \text{……………(3)}$$

   Eliminate a and b by using equations (1) (2) the eliminate of a and b is called Singular Integral.

   The general method of solving a non linear differential equation
   F(x,y,z,p,q)=0
   Is known as Char pit's Methods which will be discussed later.

Laplace Transform:

1 Introduction

Let f(t) be a given function which is defined for all positive values of t, if

$$F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

exists, then F(s) is called *Laplace transform* of f(t) and is denoted by

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\, dt$$

The inverse transform, or inverse of $\mathcal{L}\{f(t)\}$ or F(s), is

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

where *s is real or complex* value.

[Examples]

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad ; \qquad \mathcal{L}\{ e^{at} \} = \frac{1}{s - a}$$

$$\mathcal{L}\{ \cos \omega t \} = \int_0^\infty e^{-st} \cos \omega t\, dt$$

$$= \left. \frac{e^{-st} (-s \cos \omega t + \omega \sin \omega t)}{\omega^2 + s^2} \right|_{t=0}^{\infty}$$

$$= \frac{s}{s^2 + \omega^2}$$

(Note that $s > 0$, otherwise $e^{-st} |_{t=\infty}$ diverges)

$$\mathcal{L}\{\sin \omega t\} = \int_0^\infty e^{-st} \sin \omega t \, dt \qquad \text{(integration by parts)}$$

$$= \left. \frac{-e^{-st} \sin \omega t}{s} \right|_{t=0}^{\infty} + \frac{\omega}{s} \int_0^\infty e^{-st} \cos \omega t \, dt$$

$$= \frac{\omega}{s} \int_0^\infty e^{-st} \cos \omega t \, dt$$

$$= \frac{\omega}{s} \mathcal{L}\{\cos \omega t\} = \frac{\omega}{s^2 + \omega^2}$$

Note that

$$\mathcal{L}\{\cos \omega t\} = \int_0^\infty e^{-st} \cos \omega t \, dt \qquad \text{(integration by parts)}$$

$$= \left. \frac{-e^{-st} \cos \omega t}{s} \right|_{t=0}^{\infty} - \frac{\omega}{s} \int_0^\infty e^{-st} \sin \omega t \, dt$$

$$= \frac{1}{s} - \frac{\omega}{s} \mathcal{L}\{\sin \omega t\}$$

$$\Rightarrow \qquad \mathcal{L}\{\sin \omega t\} = \frac{\omega}{s} \mathcal{L}\{\cos \omega t\} = \frac{\omega}{s^2} - \frac{\omega^2}{s^2} \mathcal{L}\{\sin \omega t\}$$

$$\Rightarrow \qquad \mathcal{L}\{\sin \omega t\} = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}\{t^n\} = \int_0^\infty t^n e^{-st} \, dt \qquad \text{(let } t = z/s, \; dt = dz/s \text{)}$$

$$= \int_0^\infty \left[\frac{z}{s}\right]^n e^{-z} \frac{dz}{s} = \frac{1}{s^{n+1}} \int_0^\infty z^n e^{-z} \, dz$$

$$= \frac{\Gamma(n+1)}{s^{n+1}} \qquad \text{(Recall } \Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt \text{)}$$

If $n = 1, 2, 3, \ldots$ $\quad \Gamma(n+1) = n!$

$$\Rightarrow \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \quad \text{where n is a positive integer}$$

[Theorem] Linearity of the Laplace Transform

$$\mathcal{L}\{ a\, f(t) + b\, g(t) \} = a\,\mathcal{L}\{ f(t) \} + b\,\mathcal{L}\{ g(t) \}$$

where a and b are constants.

[Example] $\mathcal{L}\{ e^{at} \} = \dfrac{1}{s - a}$

$\mathcal{L}\{ \sinh at \} = ??$

Since

$$\mathcal{L}\{ \sinh at \} = \mathcal{L}\left\{ \frac{e^{at} - e^{-at}}{2} \right\}$$

$$= \frac{1}{2}\,\mathcal{L}\{ e^{at} \} - \frac{1}{2}\,\mathcal{L}\{ e^{-at} \}$$

$$= \frac{1}{2}\left\{ \frac{1}{s - a} - \frac{1}{s + a} \right\} = \frac{a}{s^2 - a^2}$$

[Example] Find $\mathcal{L}^{-1}\left\{ \dfrac{s}{s^2 - a^2} \right\}$

$$\mathcal{L}^{-1}\left\{ \frac{s}{s^2 - a^2} \right\} = \mathcal{L}^{-1}\left\{ \frac{1}{2}\left[\frac{1}{s - a} + \frac{1}{s + a} \right] \right\}$$

$$= \frac{1}{2}\,\mathcal{L}^{-1}\left\{ \frac{1}{s - a} \right\} + \frac{1}{2}\,\mathcal{L}^{-1}\left\{ \frac{1}{s + a} \right\}$$

$$= \frac{1}{2}\, e^{at} + \frac{1}{2}\, e^{-at} = \frac{e^{at} + e^{-at}}{2}$$

$$= \cosh at$$

Existence of Laplace Transforms

[Example] $\mathcal{L}\{ 1/t \} = ??$

From the definition,

$$\mathcal{L}\{1/t\} = \int_0^\infty \frac{e^{-st}}{t}\,dt = \int_0^1 \frac{e^{-st}}{t}\,dt + \int_1^\infty \frac{e^{-st}}{t}\,dt$$

But for t in the interval $\mathbf{0 \le t \le 1}$, $e^{-st} \ge e^{-s}$; if $s > 0$, then

$$\int_0^\infty \frac{e^{-st}}{t}\,dt \ge e^{-s}\int_0^1 \frac{dt}{t} + \int_1^\infty \frac{e^{-st}}{t}\,dt$$

However,

$$\int_0^1 t^{-1}\,dt = \lim_{A\to 0}\int_A^1 t^{-1}\,dt = \lim_{A\to 0} \ln t \Big|_A^1$$

$$= \lim_{A\to 0}(\ln 1 - \ln A) = \lim_{A\to 0}(-\ln A) = \infty$$

$\Rightarrow$ $\displaystyle\int_0^\infty \frac{e^{-st}}{t}\,dt$ diverges,

$\Rightarrow$ no Laplace Transform for 1/t !

Piecewise Continuous Functions

A function is called **piecewise continuous** in an interval $a \le t \le b$ if the interval can be subdivided into a finite number of intervals in each of which the function is continuous and has finite right- and left-hand limits.

Existence Theorem

(Sufficient Conditions for Existence of Laplace Transforms)

Let f be piecewise continuous on $t \ge 0$ and satisfy the condition

$$| f(t) | \le M e^{\gamma t}$$

for fixed non-negative constants γ and M, then

$\mathcal{L}\{ f(t) \}$

exists for all $s > \gamma$.

[Proof]

Since f(t) is piecewise continuous, $e^{-st} f(t)$ is integratable over any finite interval on $t > 0$,

$$| \mathcal{L}\{ f(t) \} | = \left| \int_0^\infty e^{-st} f(t)\, dt \right| \le \int_0^\infty e^{-st} |f(t)|\, dt$$

$$\le \int_0^\infty M e^{\gamma t} e^{-st}\, dt = \frac{M}{s - \gamma} \quad \text{if } s > \gamma$$

$\Rightarrow$ $\mathcal{L}\{ f(t) \}$ exists.

[Examples] Do $\mathcal{L}\{ t^n \}$, $\mathcal{L}\{ e^{t^2} \}$, $\mathcal{L}\{ t^{-1/2} \}$ exist?

(i) $e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots + \frac{t^n}{n!} + \ldots$

$\Rightarrow$ $t^n \le n!\, e^t$

$\Rightarrow$ $\mathcal{L}\{ t^n \}$ exists.

(ii) $e^{t^2} > M e^{\gamma t}$

$\Rightarrow$ $\mathcal{L}\{ e^{t^2} \}$ may not exist.

(iii) $\mathcal{L}\{ t^{-1/2} \} = \sqrt{\frac{\pi}{s}}$, but note that $t^{-1/2} \to \infty$ for $t \to 0$!

2. Objectives

The goal of this study is to learn

(i) How to solve the Partial differential equations by Laplace Transform

(ii) How to find the solutions

3. Methodology

By using Laplace Transform Technique how to find the solution of Partial Differential Equations.

Applications:

A function of two or more variables may also have a Laplace transform. Suppose x and t are two independent variables ;consider t as the principal variable and x as the secondary variable. When the Laplace transform is applied with t as the a variable, the PDE is reduced to an ordinary differential equation of the t-transform U(x, s),where x is the independent variable. The general solution U(x, s) of the ODE is then fitted to the BCs of the original problem. Finally ,the solution u(x, t) is obtained by using the Laplace inversion formula. Thus, the Laplace transform is specially suited to solving initial boundary value problems(IBVP), when conditions are prescribed at t=0. We have already noted situations naturally arise in case of heat conduction equation and wave equation.

Formulae :

If u(x, t) is a function of two variables x and t, then

(i) $$L\left[\frac{\partial u}{\partial t}; s\right] = sU(x,s) - u(x,0)$$

(ii) $$L\left[\frac{\partial^2 u}{\partial t^2}; s\right] = s^2U(x,s) - su(x,0) - u_t(x,0)$$

(iii) $L\left[\frac{\partial u}{\partial x}; s\right] = \frac{dU(x,s)}{dx}$

(iv) $L\left[\frac{\partial^2 u}{\partial x^2}; s\right] = \frac{d^2}{dx^2}U(x,s)$

(v) $L\left[\frac{\partial^2 u}{\partial x \partial t}; s\right] = s\frac{d}{dx}U(x,s) - \frac{d}{dx}u(x,0)$

Examples

Problem 1 : Solution of Diffusion Equation.

Solve the following IBVP using Laplace technique:

$$PDE : u_t = u_{xx,}\, 0 < x < 1, t > 0$$
$$BCs : u(x,0) = 1, u(1,t) = 1, t > 0,$$
$$IC : u(x,0) = 1 + \sin \pi x, 0 < x < 1.$$

Solution: Taking the Laplace transform of both sides of the given PDE, we have

$$sU(x,s) - u(x,0) = \frac{d^2U}{dx^2},$$

Thus, the solution of the second order PDE reduces to the solution of second order ODE given by

$\frac{d^2U}{dx^2} - sU(x,s) = -(1 + \sin \pi x)$ (after using IC).

The general solution of above equation is found to be

$U(x,s) = Ae^{\sqrt{sx}} + Be^{-\sqrt{sx}} + \frac{1}{s} + \frac{\sin \pi x}{\pi^2 + s}$ ……………………………….(1)

But, $U(0,t) = 1, u(1,t) = 1$ and their Laplace transforms are

$$U(0,s) = \frac{1}{s}, U(1,s) = \frac{1}{s}$$

From Eq (1),we have

$A+B+\frac{1}{s}=\frac{1}{s}$,Hence A+B=0 and $Ae^{\sqrt{s}}+Be^{-\sqrt{s}}+\frac{1}{s}=\frac{1}{s}$.

Therefore $Ae^{\sqrt{s}}+Be^{-\sqrt{s}}=0$

This is a homogeneous system ; the determinant of the coefficient matrix is

$$\left|\begin{pmatrix} 1 & 1 \\ e^{\sqrt{s}} & e^{-\sqrt{s}} \end{pmatrix}\right| = e^{-\sqrt{s}} - e^{\sqrt{s}} \neq 0 .$$

Thus the only possible solution is the trivial solution and therefore,

A=B=0

From Eq (1) ,we now have

$$U(x,s)=\frac{1}{s}+\frac{\sin \pi x}{\pi^2+s} \text{ ....(2)}$$

Taking the inverse Laplace transform of equation (2),we get

$$u(x,t)=L^{-1}\left(\frac{1}{s};t\right)+L^{-1}\left[\frac{\sin \pi x}{\pi^2+s};t\right]$$

Thus,

$$u(x,t)=1+\sin \pi e^{-\pi^2 t}$$

is the required solution.

Problem 2: Solution of Wave Equation

Using the Laplace transform method, solve the IBVP described as

$$PDE: u_{xx} = \frac{1}{c^2} u_{tt} - \cos\omega t, 0 \le x < \infty, 0 \le t < \infty$$
$$BCs: u(0,t) = 0;$$
$$ICs: u_t(x,0) = u(x,0) = 0.$$

Solution: Taking the Laplace transform of PDE, we obtain

$$\frac{d^2U}{dx^2} = \frac{1}{c^2}[s^2U(x,s) - su(x,0) - u_t(x,0) - \frac{s}{s^2+\omega^2}$$

Using ICs, we get

$$\frac{d^2U}{dx^2} = \frac{s^2}{c^2}U(x,s) = -\frac{s}{s^2+\omega^2}$$

Its general solution is found to be

$$U(x,s) = Ae^{\left(s/c\right)x} + Be^{-\left(s/c\right)x} + \frac{c^2}{s(s^2+\omega^2)}$$

As x tends to ∞,the transform should also be bounded which is possible if A=0,thus

$$U(x,s) = Be^{-\left(s/c\right)x} + \frac{c^2}{s(s^2+\omega^2)} \text{..........(1)}$$

Taking the Laplace transform of BC ,we get

U(0,s)=0

Using this result in Eq (1) ,we have

$$B = -\frac{c^2}{s(s^2+\omega^2)}.$$

Hence,

$$U(x,s) = \frac{c^2}{s(s^2+\omega^2)}.[1-e^{-\left(s/c\right)x}]$$

Now,taking its inverse Laplace transform,we get

$$u(x,t)=c^2L^{-1}[\frac{1}{s(s^2+\omega^2)};t]-c^2L^{-1}[\frac{e^{-\left(s/c\right)x}}{s(s^2+\omega^2)};t] \quad \ldots\ldots\ldots\ldots\ldots(2)$$

But,

$$L^{-1}[\frac{1}{s(s^2+\omega^2)};t]=\frac{1}{\omega^2}\{L^{-1}[\frac{1}{s};t]-L^{-1}[\frac{s}{s^2+\omega^2};t]\}=\frac{1}{\omega^2}(1-\cos\omega t),$$

$$L^{-1}[\frac{e^{-\left(x/c\right)s}}{s(s^2+\omega^2)};t]=\frac{1}{\omega^2}\{(1-\cos\omega(t-\frac{x}{c})\}H(t-\frac{x}{c}),$$

Where H is the Heaviside unit function. Substituiting these results in Eq (2), we arrive at

$$u(x,t)=\frac{c^2}{\omega^2}(1-\cos\omega t)-\frac{c^2}{\omega^2}[(1-\cos\omega(t-\frac{x}{c})H(t-\frac{x}{c})]$$

Which is the required solution.

Problem 3: Solve the IBVP described by

$$PDE: u_{tt}=u_{xx}, 0<x<1, t>0$$

$$BCs: u(0,t)=u(1,t)=0, t>0$$

$$ICs: u(x,0)=\sin\pi x, u_t(x,0)=-\sin\pi x, 0<x<1$$

Solution: Taking the Laplace transform of the given PDE,we get

$$\frac{d^2U}{dx^2}=s^2U(x,s)-su(x,0)-u_t(x,0)$$

Using the intial conditions,this equation becomes

$$\frac{d^2U}{dx^2}=s^2U=(1-s)\sin\pi x$$

Its general solution is found to be

$$U(x,s) = Ae^{sx} + be^{-sx} + \frac{(s-1)\sin \pi x}{\pi^2 + s^2} \quad \ldots\ldots\ldots(1)$$

The Laplace transform of the BCs gives

$$U(0,s) = 0, U(1,s) = 0 \quad \ldots\ldots\ldots(2)$$

Using Eq (2) into Eq.(1) ,we find A=B=0.Hence ,we obtain

$$U(x,s) = \frac{(s-1)\sin \pi x}{\pi^2 + s^2}$$

Taking the inverse Laplace transform ,we get

$$u(x,t) = \sin \pi x\{L^{-1}[\frac{s-1}{s^2+\pi^2};t]\} = \sin \pi x\{L^{-1}[\frac{s}{s^2+\pi^2};t] - L^{-1}[\frac{1}{s^2+\pi^2};t]\}$$

$$= \sin \pi x(\cos \pi t - \frac{\sin \pi t}{\pi})$$

Hence the required solution of the given IBVP is

$$u(x,t) = \sin \pi x(\cos \pi t - \frac{\sin \pi t}{\pi}).$$

Conclusion:

Application of Laplace Transforms to Partial Differential Equations , we illustrated the effective use of Laplace transforms in solving differential equations. The transform replaces a differential equation in y(t) with an algebraic equation in its transform ˜y(s). It is then a matter of finding the inverse transform of ˜y(s) either by partial fractions and tables or by residues . Laplace transforms also provide a potent technique for solving partial differential equations. When the transform is applied to the variable t in a partial differential equation for a function y(x, t), the result is an ordinary differential equation for the transform ˜y(x, s). The ordinary differential equation is solved for ˜y(x, s) and the function is inverted to yield y(x, t). We illustrate this procedure with three physical examples. The first two examples are on unbounded spatial intervals; inverse transforms are found in tables. The last three examples are on bounded spatial intervals; inverse transforms are calculated with residues.

References

1. Introduction to Partial Differential Equations by K.Sankar Rao. Prantice Hall India Learning Pvt.Ltd
2. Kreyzing, Erwin , 1993, *Advanced Engineering Mathematics*. 7th Edition, Wayne.
3. A.C. King, J. Billingham and S.R.Otto, 2003, *Differential equations (Linear, Non linear,Ordinary , partial)*, Cambridge University Press.
4. George F. Simmons, *Differential Equation with Applications and Historical notes, 2nd* edition, Tata Mcgraw Hill education private limited.