COMMISSIONERATE OF COLLEGIATE EDUCATION

"JIGNASA"

STUDENTS' STUDY PROJECT

2021-22

DEPARTMENT OF MATHEMATICS

KAKATIYA GOVERNMENT COLLEGE

HANUMAKONDA

JIGNASA; STUDENTS' STUDY PROJECT.

ON

"Golden Ratio and Its Applications".

PARTICIPANTS

A. ARUN B.SC (MPC) III YR.

CH. MANASWITHA, B.SC (MPC) III YR.

P. MAHENDAR, B.SC (MPC) III YR

E. SHARAVANI, B.SC (MSTCS) III YR.

K. SANDEEP, (MSTCS) II YR

MENTORS

1. **M.VENUGOPAL, Asst. Prof of Mathematics.**
2. **M. RADHIKA, Asst. Prof of Mathematics**

DECLARATION

We, the undersigned students declare that the project entitled, **"Golden Ratio and Its Applications"** Submitted to the Commissioner of Collegiate Education, Telangana under JIGNASA is our original work.

PARTICIPANTS

A. ARUN B.SC (MPC) III YR.

CH. MANASWITHA, B.SC (MPC) III YR.

P. MAHENDAR, B.SC (MPC) III YR

E. SHARAVANI, B.SC (MSTCS) III YR.

K. SANDEEP, (MSTCS) II YR

CERTIFICATE FROM THE MENTOR

This is to certify that the project entitled, "**Golden Ratio and Its Applications"** is a bonafied record of independent work done by the students under our supervision. It is submitted to the Commissioner of Collegiate Education, Telangana under JIGNASA 2021-22.

MENTORS

1. M.VENUGOPAL

2. M. RADHIKA

Department of Mathematics

KGC, Hanumakonda.

OUTLINE

Golden Ratio and its Applications

1. Introduction:

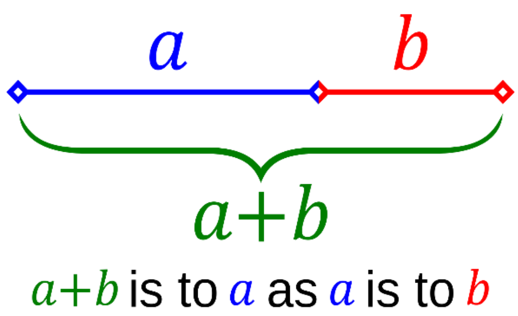
The **Golden Ratio** is a mathematical ratio which is **also** known as the **golden section, golden mean**, or **divine proportion**. It is the irrational number $(1 + \sqrt{5})/2$, often denoted by the Greek letter φ which is approximately equal to 1.618. It is commonly found in nature, and when used in a design, it fosters organic and natural-looking compositions that are aesthetically pleasing to the eye. According to Mario Livio, an Israeli-American Astrophysicist Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece, through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler, to present-day scientific figures such as Oxford physicist Roger Penrose, have spent endless hours over this simple ratio and its properties. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.

It is the ratio of a line segment cut into two pieces of different lengths such that the ratio of the whole segment to that of the longer segment is equal to the ratio of the longer segment to the shorter segment. The ancient Greeks recognized this "dividing" or "sectioning" property, a phrase that was ultimately shortened to simply "the section." It was more than 2,000 years later that both "ratio" and "section" were designated as "golden" by German mathematician Martin Ohm in 1835. The Greeks also had observed that the golden ratio provided the most aesthetically pleasing proportion of sides of a rectangle, a notion that was enhanced during the Renaissance by, for example, the work of the Italian polymath Leonardo da Vinci and the publication of *De divina proportione* (1509; *Divine Proportion*), written by the Italian mathematician Luca Pacioli and illustrated by Leonardo.

The golden ratio occurs in many mathematical contexts. It is geometrically constructible by straightedge and compass, and it occurs in the investigation of the Archimedean and Platonic solids. It is the limit of the ratios of consecutive terms of the Fibonacci number sequence 1, 1, 2, 3, 5, 8, 13… in which each term beyond the second is the sum of the previous two. In modern mathematics, the golden ratio occurs in the description of fractals, figures that exhibit self-similarity and play an important role in the study of chaos and dynamical systems.

1.1 Definitions

Golden Ratio: Two numbers are said to be in golden ratio if their <u>ratio</u> is the same as the ratio of their <u>sum</u> to the larger of the two quantities. Expressed algebraically, for any two numbers $a > 0, b > 0$ if $\frac{a+b}{a} = \frac{a}{b}$, then *a*, *b* are said to be in golden ratio. This ratio is usually denoted by Greek letter φ (phi). It is an irrational number that is a solution to the quadratic equation $x^2 - x - 1 = 0$ with a value of $\varphi = \frac{1+\sqrt{5}}{2}$ =1.618033988749....



(Source: Google image)

Calculation:

Let the two quantities a and b are in the *golden ratio* φ, then

$$\frac{a+b}{a} = \frac{a}{b} = \varphi$$

One method for finding the value of φ is to start with the left fraction. Through simplifying the fraction and substituting in $\frac{b}{a} = \frac{1}{\varphi}$,

$$\frac{a+b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\varphi} = \varphi$$

Therefore,

$$1 + \frac{1}{\varphi} = \varphi .$$

Multiplying by φ gives $\varphi + 1 = \varphi^2$, Which can be rearranged to

$\varphi^2 - \varphi - 1 = 0.$

Using the quadratic formula, two solutions are obtained:

$\frac{1+\sqrt{5}}{2}$ =1.618033988749.... and $\frac{1-\sqrt{5}}{2}$ = -0.618033988749....

Because φ is the ratio between positive quantities, it is necessarily the positive one.

Therefore $\frac{1+\sqrt{5}}{2}$ =1.618033988749.... is the golden ratio.

Sequence: A function $s : N \to \mathbb{R}$ is called as a sequence with terms $s_1, s_2, s_3, ...$ as the images of 1, 2, 3… respectively. Usually it is denoted by $< s_n >$.

Ex: $s_n = \frac{1}{n}, t_n = \frac{n+1}{n}, ...$ etc.

Fibonacci sequence:

A sequence $< F_n >$ defined by the recursive formula as given below is known as Fibonacci sequence.

$F_1 = 1, F_2 = 1, F_n = F_{n-2} + F_{n-1} \ \forall\, n \geq 3.$

The Fibonacci numbers were first described in Indian mathematics as early as 200 BC in work by Pingala on enumerating possible patterns of Sanskrit poetry formed from syllables of two lengths. They are named after the Italian mathematician Leonardo of Pisa, later known as Fibonacci, who introduced the sequence to Western European mathematics in his 1202 book *Liber Abaci*.

Relationship to Fibonacci sequence

The mathematics of the golden ratio and of the Fibonacci sequence are intimately interconnected. The Fibonacci sequence is:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233…,

Consider the following ratios of terms of the above sequence to its predecessor as given below

$$\frac{1}{1}=1,$$

$$\frac{2}{1}=2,$$

$$\frac{3}{2}=1.5,$$

$$\frac{5}{3}=1.6666....,$$

$$\frac{8}{5}=1.6,$$

$$\frac{13}{8}=1.625,$$

$$\frac{21}{13}=1.615,$$

$$\frac{34}{21}=1.619047...,$$

$$\frac{55}{34}=1.61764....,$$

$$\frac{89}{55}=1.618181818...,$$

$$\frac{144}{89}=1.617977....,$$

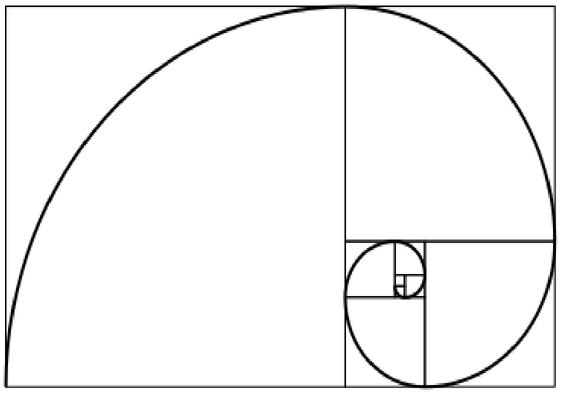
$$\frac{233}{144}=1.618055...$$

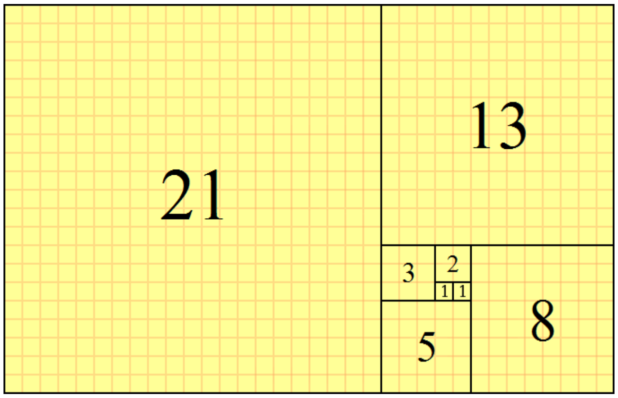
Proceeding in a similar way, we see that $\lim \frac{F_{n+1}}{F_n}=\varphi=1.618033988749....$ ....which is the golden ratio.

Golden spiral-Fibonacci Spiral:

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio. That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. A Fibonacci spiral which approximate the golden spiral using Fibonacci sequence square sizes up to 34. The spiral is drawn starting from the inner 1 X 1 square and continues outwards to successively larger squares.

(a) (b)





Figures: (a) Golden spiral (b) Fibonacci spiral (Source: Wikipedia)

The Golden Spiral can be used as a guide to determine the placement of content. Our eye is naturally drawn to the center of the spiral, which is where it will look for details, so focus your design on the center of the spiral and place areas of visual interest within the spiral.

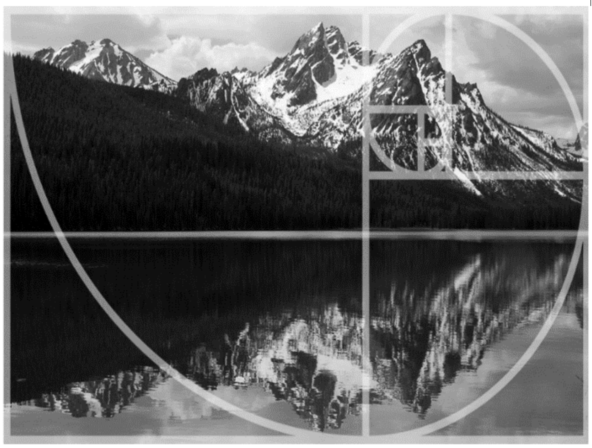
2. Applications and observations of Golden Ratio:

(i) Nature

Many patterns in nature including ferns, flowers, seashells, galaxy, even hurricanes, which perhaps why we find it so visually appealing. Because it is, indeed, nature at its finest golden ratio. All these are in pattern of golden spiral.

(ii) Golden ratio in photography

The Fibonacci spiral is one of the main ways photographers can use the golden ratio in photography. Many famous photographers are known for their use of the golden ratio in photography few of them are Ansel Adams and Henri Cartier- Bresson who used it often in their landscape portraits that they captured.



(c)

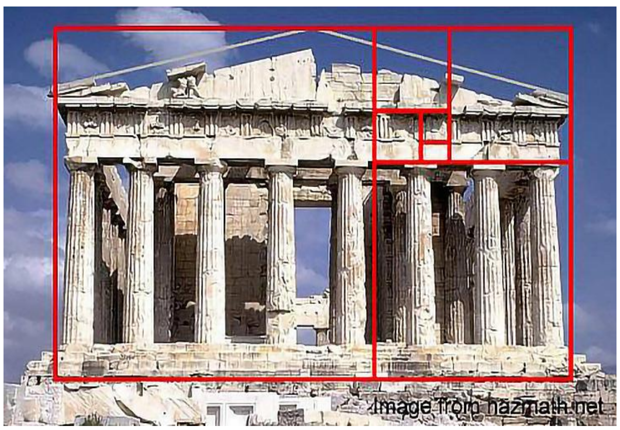
(d)

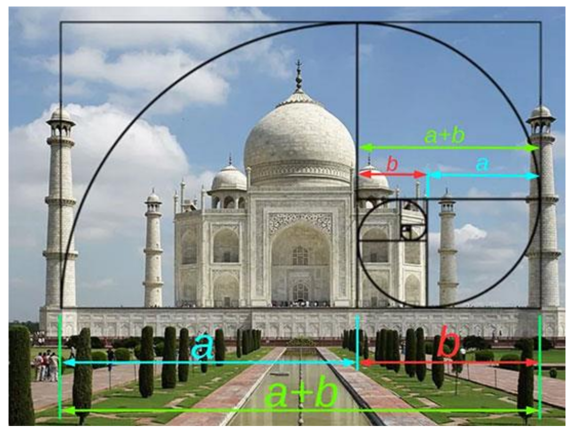
Figures: (c) By Ansel Adams (d) By Henri Cartier- Bresson (Google Images)

(iii) Golden ratio in Architecture

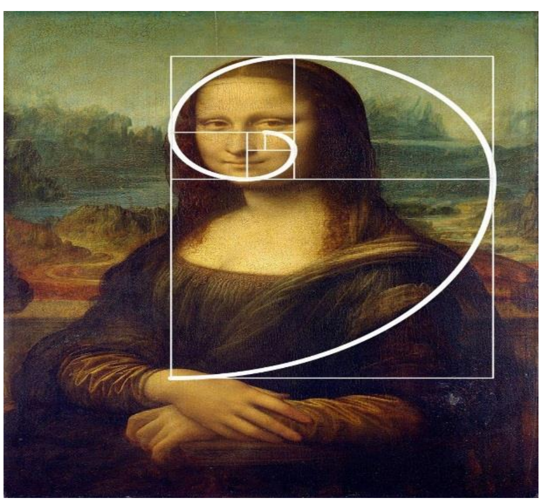
The Greek sculptor Phidias sculpted many things including the bands of sculpture that run above the columns of the Parthenon (Figure (e)). Even from the time of the Greeks, a rectangle whose sides are in the "golden proportion" has been known since it occurs naturally in some of the proportions of the Five Platonic. This rectangle is supposed to appear in many of the proportions of that famous ancient Greek temple in the Acropolis in Athens, Greece. The Taj Mahal in India was also constructed using the Golden Ratio (Figure (f)). The main building of the Taj Mahal was designed using the Golden Ratio. This is why it looks so perfect. The rectangles that served as the basic outline for the exterior of the building were all in the Golden Proportion. The Ahmes papyrus of Egypt gives an account of the building of the Great Pyramid of Giaz in 4700 B.C. with proportions according to a "sacred ratio."

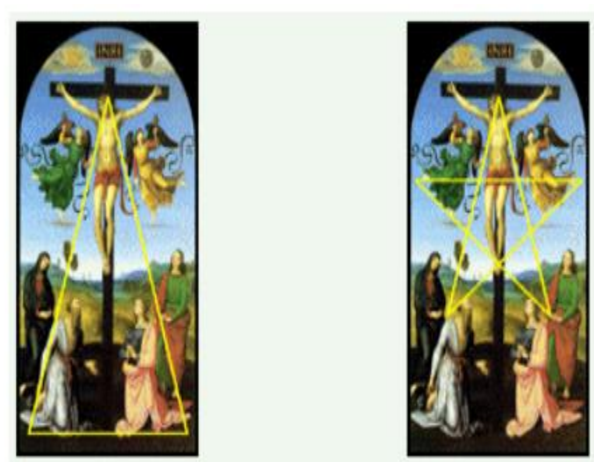
(iv) **Golden ratio in Art:** Leonardo Da Vinci created the illustrations for "De Divina Proportione" (On the Divine Proportion), a book about mathematics written by Luca Pacioli around 1498 and first published in 1509. In the book, Pacioli writes about mathematical and artistic proportion, particularly the mathematics of the golden ratio and its application in art and architecture (Figure (g)). The book contains dozens of beautiful illustrations of three-dimensional geometric solids and templates for script letters in calligraphy. The picture (h) ***Crucifixion*** *by Raphael* is a well-known example, in which we can find a Golden Triangle and also Pentagram. In this picture, a golden triangle can be used to locate one of its underlying pentagrams.





(e) (f)





(g) (h)

(Source: Google Images)

(v) **Golden ratio in Music**

Though ancient Chinese, Indians, Egyptians and Mesopotamians are known to have studied the mathematical principles of sound, the Pythagoreans (in particular Philolaus and Archytas) of ancient Greece were the first researchers known to have investigated the expression of musical scales in terms of numerical ratios, particularly the ratios of small integers. Their central doctrine was that "all nature consists of harmony arising out of numbers”.

The Fibonacci series appears in the foundation of aspects of art, beauty and life. Even music has a foundation in the series, as:

- There are 13 notes in the span of any note through its octave.
- A scale is composed of 8 notes, of which the
- 5th and 3rd notes create the basic foundation of all chords, and
- are based on a tone which are combination of 2 steps and 1 step from the root tone, that is the 1st note of the scale.

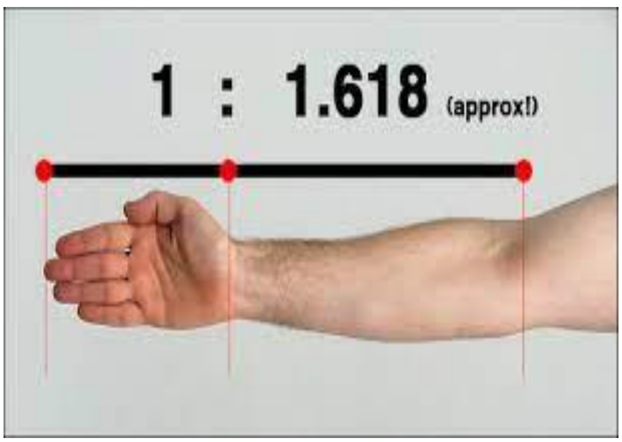
Princeton University Professor and Field Medal winner Dr Manjul Bhargava gave an enthralling lecture at the Madras Music Academy on the relation between mathematics and music. This presentation revolved around the rhythms of ancient Indian poetry, music and the math behind them as expounded in various treatises such as the *Nātyashāstra* and the *Sangeeta Ratnakara*. He explained how even fundamental concepts in math could be understood in terms of poetry and rhythm thus demonstrating that the subtle yet vital connection between the poetry, the performing arts and mathematics. It enthralled every one, when he composed Tabla beats using Fibonacci numbers.

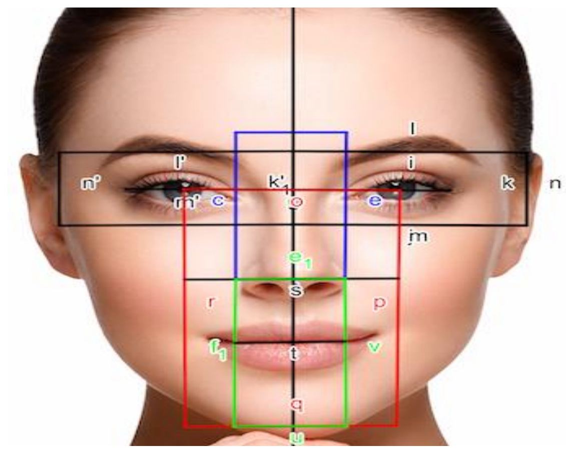
(vi) **Golden ratio in Human body**

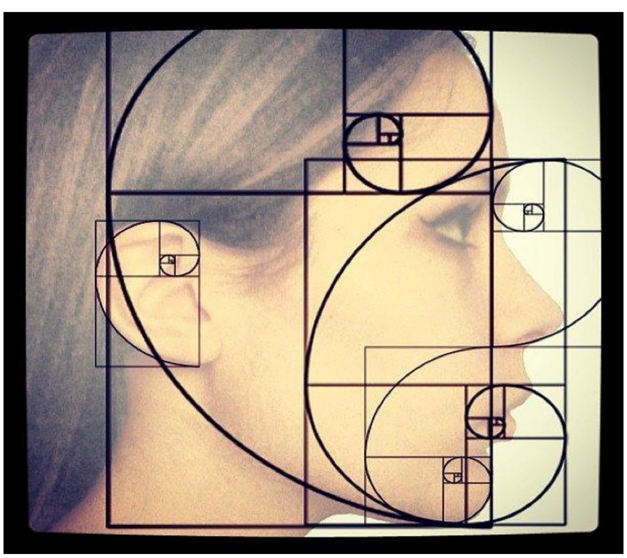
In a study investigating whether skull shape follows the Golden Ratio (1.618 …), Johns Hopkins researchers compared 100 human skulls to 70 skulls from six other animals, and found that the human skull dimensions followed the Golden Ratio. The skulls of less related species such as dogs, two kinds of monkeys, rabbits, lions and tigers, however, diverged from this ratio.

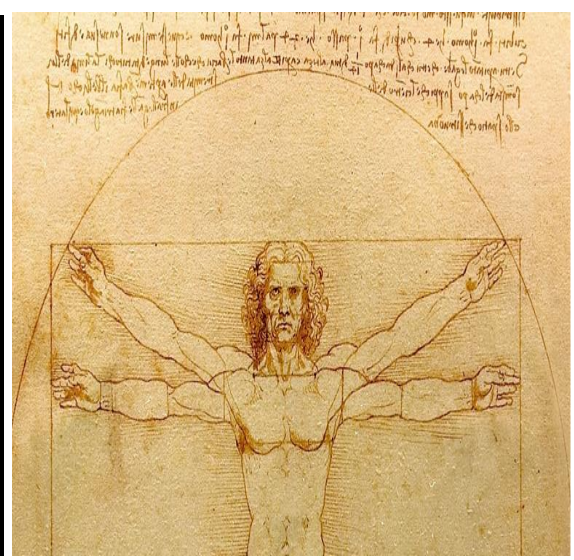
“The other mammals we surveyed actually have unique ratios that approach the Golden Ratio with increased species sophistication,” says Rafael Tamargo, M.D., professor of neurosurgery at the Johns Hopkins University School of Medicine. “We believe that this finding may have important anthropological and evolutionary implications.”

Above research demonstrates how golden ratio plays important role in human life too. Model Bella Hadid has been named as the most beautiful woman in the world according to Golden Ratio of Beauty Phi. Her face is considered 94.35% perfect. Amber Heard, Bella Hadid and Beyoncé have the most 'perfect' faces in the world. The following figures demonstrates the ratio of body parts that should to be in golden ration in order to be in perfect shape and good looking.









Source: Google images

3. Aim and Objectives of the study

The aim of the present study is to analyze and understand the Golden ratio, and its connection with aesthetic.

The objectives of this project are

- To find the relation between Golden ratio and Fibonacci numbers (Sequence).
- To analyze the connection of golden ratio with nature, human body, art, architecture, music and photography.
- To understand spiral patterns of nature and its connection with golden ratio.
- To collect some flowers of botanical garden of our college K.G.C Hanumakonda, and examine whether they fit in golden ratio.
- To examine whether faces of students of our college are as per golden ratio.

4. Review of the literature

Ancient Greek mathematicians first studied what we now call the golden ratio, because of its frequent appearance in geometry , the division of a line into "extreme and mean ratio" (the golden section) is important in the geometry of regular pentagrams and pentagons. According to one story, 5th-century BC mathematician Hippasus discovered that the golden ratio was neither a whole number nor a fraction (an irrational number), surprising Pythagoreans. Euclid's Elements (c. 300 BC) provides several propositions and their proofs employing the golden ratio [1], and contains its first known definition which proceeds as follows [2]: A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser [1]. The golden ratio was studied peripherally over the next millennium. Abu Kamil (c. 850–930) employed it in his geometric calculations of pentagons and decagons; his writings influenced that of Fibonacci (Leonardo of Pisa) (c. 1170–1250), who used the ratio in related geometry problems although he never connected it to the series of numbers named after him [3]. Luca Pacioli named his book Divina proportione (1509) after the ratio, and explored its properties including its appearance in some of the Platonic solids [4]. Leonardo da Vinci, who illustrated the aforementioned book, called the ratio the section aurea ('golden section'). 16th-century mathematicians such as Rafael Bombelli solved geometric problems using the ratio [5].German mathematician Simon Jacob (d. 1564) noted that consecutive Fibonacci numbers converge to the golden ratio [6]; this was rediscovered by Johannes Kepler in 1608[1]. The first known decimal approximation of the (inverse) golden ratio was stated in 1597 by Michael Maestlin of the University of Tübingen in a letter to Kepler, his former student [7]. The same year, Kepler wrote to Maestlin of the Kepler triangle, which combines the golden ratio with the Pythagorean Theorem. Kepler said of these: Geometry has two great treasures: one is the theorem of Pythagoras, the other the division of a line into extreme and mean ratio. The first we may compare to a mass of gold, the second we may call a precious jewel [8]. By 1910, mathematician Mark Barr began using the Greek letter Phi (φ) as a symbol for the golden ratio [9]. The zome construction system, developed by Steve Baer in the late 1960s, is based on the symmetry system of the icosahedron/dodecahedron, and uses the golden ratio ubiquitously. Between 1973 and 1974, Roger Penrose developed Penrose tiling, a pattern related to the golden ratio both in the ratio of areas of its two rhombic tiles and in their relative frequency within the pattern [10]. This led to Dan Shechtman's early 1980s discovery of quasicrystals, some of which exhibit icosahedral symmetry [11, 12].

5. Methodology

For this student study project, Learning by doing method is adopted. Students will understand what exactly golden ratio is and its importance in day to day life. During this project work, students will observe flowers that are visually appealing in our college botanical garden, and see whether their pattern follow the golden spiral and number of petals is a Fibonacci number. Also, students of this project work will measure the dimensions of faces, arms, and fingers of few of their classmates to examine whether they fit in golden ratio.

6. Data Collection

- During this study project, we observed certain things in premises of the college
- We collected some flowers from botanical garden of the college.
- Randomly measured dimensions of faces of 15 students and
- Measurements of hand

7. Findings

- Out of 6 flowers , number of petals of 3 flowers is a Fibonacci number (5) and one flower's petals are spiral in nature(Crown of thorn).
- Leaves of one plant pattern is spiral.
- Out of 15 students, 7 students' face and hand measurements are very close to the golden ratio which ranges between 1.58 to 1.64 .

8. Conclusions

- Golden ratio and its relationship with Fibonacci sequence is studied through experiential learning during this study project.
- Despite myths and misconceptions about how golden ratio is related to aesthetically pleasing phenomenon, it is fact that golden ratio is interconnected with Fibonacci sequence.
- The Fibonacci sequence certainly does appear in nature as it is both linked to the way that populations grow, and also to the way that shapes can be fitted together. For example, the sequence can be seen in the spirals on sun flowers which have to fit together in an ordered fashion, and in the leaves on some plants that need to be arranged to capture the most sunlight. As a result it is possible to observe ratios close to the golden ratio arising in certain natural phenomena.

References

1. Livio, Mario (2003). *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number.* New York, Broadway Books. ISBN 978-0-7679-0816-0.

2. Hemenway, Priya (2005). *Divine Proportion: Phi in Art, Nature, and Science*. New York: Sterling. pp. 20–21. ISBN 978-1-4027-3522-6.

3. Richard Fitzpatrick (translator) (2007). *Euclid's Elements of Geometry.* p. 156. ISBN 978-0615179841.

4. Pacioli, Luca. *De divina proportione*, Luca Paganinem de Paganinus de Brescia (Antonio Capella) 1509, Venice.

5. Livio, Mario (2003). *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number* .p.141.New York, Broadway Books.
6. Schreiber, Peter (1995). *A Supplement to J. Shallit's Paper Origins of the Analysis of the Euclidean Algorithm*. Historia Mathematica. **22** (4): 422–424. doi:10.1006/hmat.1995.1033
7. "The Golden Ratio". The MacTutor History of Mathematics archive. Retrieved 2007-09-18.
8. Fink, Karl; Beman, Wooster Woodruff; Smith, David Eugene (1903). *A Brief History of Mathematics*: An Authorized Translation of Dr. Karl Fink's Geschichte der Elementar-Mathematik.
9. Posamentier, Alfred S.; Lehmann, Ingmar (2011). *The Glorious Golden Ratio*. Prometheus Books. p. 285. ISBN 9-781-61614-424-1.
10. Gardner, Martin (2001). *The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems.* W.W. Norton & Company. pp. 77, 88. ISBN 978-0393020236.
11. Gerlin, Andrea (October 5, 2011). *"Tecnion's Shechtman Wins Nobel in Chemistry for Quasicrystals Discovery*". Bloomberg. Archived from the original on December 5, 2014. Retrieved January 4, 2019.
12. Jaric, Marko V. (2012), *Introduction to the Mathematics of Quasicrystals*, Elsevier, p. x, ISBN 978-0323159470.
13. Wikipedia
14. Google
15. Britanica